IMU 2023 Discrete Mathematics section: Titles & Abstracts

September 2023

Yuval Filmus (Technion):

Irreducible subcube partitions

Abstract

A subcube partition is a partition of $\{0,1\}^n$ into axis-aligned subcubes. A partition is irreducible if the union of no nontrivial subpartition is itself a subcube. How small can an irreducible subcube partition be? How large? Do their exist irreducible subcube partitions consisting only of edges? What if we replace "subcube" with "affine subspace"?

Joint work with Edward Hirsch (Ariel), Sascha Kurz (Bayreuth), Ferdinand Ihringer (Gent), Artur Riazanov (EPFL), Alexander Smal (Technion), Marc Vinyals (Auckland, NZ).

Orit Raz (The Hebrew University):

On a Continuous Analogue of the Elekes–Ronyai Theorem and Related Problems

Abstract

If f is a real polynomial and A and B are finite sets of real number, each of cardinality n, then Elekes and Ronyai proved that either $f(A \times B)$ is much larger than n, or f has a very specific form (essentially, $f(x,y) = x + y$). In the talk I will tell about an analogue of this problem, where A and B are now infinite subsets of $[0,1]$, each of Hausdorff dimension α. In a recent result, joint with Josh Zahl, we prove that in this case $f(A \times B)$ will have Hausdorff dimension at least $\alpha + c$, where $c = c(\alpha) > 0$, unless f has the specific special form as above. I will explain a connection between this problem and a projection theorem, in the spirit of Kaufman, Bourgain, and Shmerkin.

Based on a joint work with Josh Zahl.

Misha Muzychuk (Ben-Gurion University):

On Jordan schemes

Abstract

Motivated by the problems appearing in the theory of experimental designs, Bose and Mesner introduced in 1959 a special class of matrix algebras, known nowadays as *Bose-Mesner* algebras of symmetric association schemes. In the same year Shah published the paper where he proposed a more general idea: to replace the standard matrix product with the Jordan product. So, in fact, he introduced the objects called later *Jordan schemes* by Cameron. While the ideas of Bose and Mesner led to a new direction in algebraic graph theory called later *algebraic combinatorics* by Bannai and Ito, Shah's idea was not developed at all. Only in 2004 Shah's approach was analyzed by Bailey. She observed that the symmetrization of any association scheme is a Jordan scheme, which led to the following question posed by Cameron: "Are there any others?".

In the talk an affirmative answer to this question will be given by providing several infinite series of proper Jordan schemes, i.e. those which do not appear via symmetrization of association schemes.

Joint work with Sven Reichard and Mikhail Klin.

Elad Aigner–Horev (Ariel University):

Smoothed analysis across Combinatorics

Abstract

By *smoothed analysis* one means the study of the properties of randomly perturbed structures. The term was coined by Spielman and Teng in 2001 and ever since then has flourished into various branches of Mathematics and Computer Science.

Being the last talk of the day, we aim at providing a survey talk pertaining to the developments of smoothed analysis in the realm of Combinatorics seen in the last 2-3 years or so; this whilst highlighting our own results in this venue. This, in particular, will take us through topics such as the discrepancy of randomly perturbed matrices (a smoothed version of the Komlós conjecture), the emergence of large complete minors in randomly perturbed graphs, the singularity of matrices perturbed by sparse (combinatorial) random matrices, and the Ramsey properties of randomly perturbed hypergraphs.

We shall linger a tad longer on the Ramsey properties of randomly perturbed hypergraphs. Here, we shall present a new variant of the strong hypergraph regularity lemma; one which houses both the strong and the weak hypeargraph regularity lemmas under one roof, so to speak; a new adequate tuple lemma will be presented and a glimpse into the merits of these will be provided.

The talk is based on joint works with subsets of Dan Hefetz, Michael Krivelevich, Yury Person, Mathias Schacht, and Michael Trushkin.
