The strongest colorings

Speaker: Jing Zhang (BIU), 14:00

Abstract: The coloring principle $Pr_1(...)$ is extracted from the study on the productivity of the chain condition for posets or topological spaces. It dated back to Galvin's work on the productivity of the countable chain condition in the 80's and was made explicit by Shelah. In the 90's, Shelah proved an instance of $Pr_1(...)$ in ZFC which implies there are $\aleph_2$-c.c posets whose product is not $\aleph_2$-c.c. He asked whether one could prove in ZFC the strongest instance of $Pr_1(...)$. We will discuss recent progresses on this problem. In particular, we show that the stick principle at $\aleph_2$, which is a weak consequence of $2^{\aleph_1} = \aleph_2$, implies the strongest instance of $Pr_1(...)$ at $\aleph_2$. The limitations and extents of $Pr_1(...)$ at inaccessible and singular cardinals will also be discussed. Joint work with Assaf Rinot.

The modal logic of (σ-centered) forcing

Speaker: Ur Ya'ar (HUJI), 15:00

Abstract: Modal logic is used to study various modalities - ways in which statements can be true, for example being necessary, or possible. In set-theory, we may consider a statement as necessary if it holds in any forcing extension of the world, and possible if it holds in some forcing extension. One can now ask what are the modal principles which capture this interpretation, or in other words - what is the "Modal Logic of Forcing"? We can also restrict ourselves only to a certain class of forcing notions, or to forcing over a specific universe, resulting in an abundance of questions to be resolved. In this talk we will present some of the tools developed by Joel Hamkins and Benedikt Loewe to answer these questions, and then focus on the modal logic of the class of σ-centered forcing notions, and a few related classes as well.

When Sierpiński met Ulam

Speaker: Tanmay Inamdar (BIU), 15:40

Abstract: Motivated by applications in club guessing we introduce a family of colouring principles $\mathsf{onto}(J, \theta)$ and $\mathsf{unbounded}(J, \theta)$ where J is a non-trivial ideal on some infinite cardinal λ and θ is a cardinal less than or equal to λ. A similar principle was considered by Sierpiński, the so-called onto mapping principle. We show that instances of these colouring principles are equivalent to the existence of an Ulam matrix on a cardinal. We also use these principles to characterise large cardinals. We give some other applications. Joint work with Assaf Rinot.

Ordinal Definable subsets of ordinals and the failure of the SCH

Speaker: Alejandro Poveda (HUJI), 16:20

A remarkable result of Shelah [1] says that if κ is a singular strong limit cardinal of uncountable cofinality then there is $x \subseteq \kappa$ such that $\mathcal{P}(\kappa) \subseteq \mathrm{HOD}_x$. That is, every subset of κ is definable by using ordinals and x as parameters. Philosophically speaking this means that, modulo an additional parameter (i.e., x), the HOD-hierarchy faithfully resembles the combinatorics of κ. In this respect, in [2] the authors showed that the assumption that κ has uncountable cofinality is necessary. Specifically, they construct a model of ZFC where κ is a singular strong limit cardinal of countable cofinality and $(\kappa^+)^{\mathrm{HOD}_x} < \kappa^+$ for all $x \subseteq \kappa$. The forcing notion responsible of yielding this configuration is a *pseudo extender based forcing* similar to the classical Extender Based Prikry forcing. Due to this reason, Sinapova later asked [3] whether it is possible to add the failure of the SCH to this picture.

In this talk we will answer affirmatively this question and discuss some related open problems. Joint work with Yair Hayut and Menachem Magidor.

References

[1] Shelah, S. (1997). Set theory without choice: not everything on cofinality is possible. Archive for Mathematical Logic, 36(2), 81-125.
[2] Cummings, J., Friedman, S. D., Magidor, M., Rinot, A., & Sinapova, D. (2018). Ordinal definable subsets of singular cardinals. Israel Journal of Mathematics, 226(2), 781-804.
[3] Sinapova, D. (2017). Oberwolfach Report No. 11/2017.

The Automorphism Tower of a Group

Speaker: Tzoor Plotnikov (HUJI), 17:00

Abstract: We will talk about the operation of forming the automorphism tower over a certain group. Namely, looking at the automorphism group of a certain group, on the automorphism group of that group, and so forth, continuing transfinitely. In the late 80's Simon Thomas has showed that for every centerless group G, the automorphism tower of G stabilizes in fewer than $\left(2^{|G|}\right)^{+}$ many steps. The question of when the tower stabilizes has been studied by Thomas, Shelah, Just, Hamkins, Fuchs, Lücke and more, and turned out to have a lot of set theoretical content. We will review the different theorems and consistency results existing, and sample some of the proofs and techniques used in the subject.