Parallel Session on Set Theory and Logic
IMU Annual Meeting 2022, Be'er-sheva

Colorings of Abelian groups

Ido Feldman (BIU), 14:10-14:45

In 1974, Hindman proved that considering the group $(\mathbb{N}, +)$, for any partition $\mathbb{N} = S_0 \uplus S_1$, there exists an infinite $X \subseteq \mathbb{N}$ such that the set of its finite sums, is monochromatic, that is, contained in one of the cells.

In contrast to this, in 2016 Komjáth showed that, for the group $(\mathbb{R}, +)$ there exists a partition $\mathbb{R} = S_0 \uplus S_1$ such that, whenever $X \subseteq \mathbb{R}$ is uncountable, not only is the set of finite sums not monochromatic, but already the set $\mathrm{FS}_2(X) := \{x + y \mid \{x, y\} \in [X]^2\}$ is not monochromatic.

These results motivate a general investigation of anti-Ramsey theory for infinite Abelian groups in the spirit of the classical partition calculus, and which in fact for some cases are a strengthening of the classical partition calculus.

Our main result is a consistency of group-theoretic generalization of a strong colouring result for triples due to Todorčević from 1994. For this, we use a principle introduced by Fernández-Bretón and Rinot which translates between the classical anti-Ramsey theory for sets, and the anti-Ramsey theory for Abelian groups.

Additive reducts of real closed fields and strongly bounded structures

Hind Abu Saleh (Haifa), 14:45-15:20

Given a real closed field R, a reduct M of a linearly ordered structure $(R, <, \ldots)$ is called strongly bounded if every M-definable subset of R is either bounded or co-bounded in R. We investigate strongly bounded additive reducts of o-minimal structures and as a corollary we prove on additive reducts of real closed fields.

Between Whitehead groups and uniformization

Mark Poór (HUJI), 15:25-16:00

By the celebrated result of S. Shelah it is independent of ZFC whether abelian groups of power the first uncountable cardinal having the Whitehead property (i.e. no nontrivial group extension by the integers) are necessarily free. In fact the existence of Whitehead groups which are non-free is equivalent to the existence of some ladder system on some stationary set of countable ordinals that admits 2-uniformization, moreover, under Martin's axiom and large continuum a group G satisfying some natural strengthening of almost freeness is automatically Whitehead. We prove that assuming enough uniformization each strongly almost free group of cardinality of the first uncountable cardinal is a Whitehead group, answering a question of Eklof and Mekler. This is joint work with S. Shelah.

Complex polynomials up to interdefinability

Benjamin T. Castle (BGU), 16:15-16:50

Motivated by recent progress toward Zilber's Restricted Trichotomy Conjecture, we study reducts of the complex field up to interdefinability over parameters. Precisely, for this talk we will focus on structures of the form $(\mathbb{C}, P_1, ..., P_n)$, where the P_i are polynomial maps of potentially different arities. Somewhat surprisingly, our main result states that almost all such structures (in a precise sense) are interdefinable. The proof uses a mix of tools from geometric stability theory, combinatorics, and algebraic geometry. This is joint work with Chieu-Minh Tran.

Hierarchies of δ-strongly compact cardinal

Zhixing You (BIU), 16:50-17:25

Questions of compactness are the most fundamental problems in modern mathematics. We know that many compactness properties follow from strong compactness, but for some of them, we don't know the exact large cardinal strength. Recently, Bagaria and Magidor introduced a weak version of strong compactness:δ-strong compactness. It turns out that δ-strong compactness can characterize some compactness properties successfully.

In this talk, we will introduce related background, and separate these δ-strong compactness for different δ simultaneously. These results reveal that many compactness properties related to δ-strong compactness have different large cardinal strength, and also answered some open questions of Bagaria and Magidor, and Boney, Unger and Brook Taylor. This is joint work with Jiachen Yuan.