IMU 2024 meeting, non-commutative algebra session.

Titles&abstracts

Louis Rowen:

Title: Tensor Products of Residue Structures

Abstract:

Given a ring R and a subgroup G of $(R,+)$, one might ask about the structure of the residue R/G. In order for the cosets to be a ring,

G must be an ideal of R.

What happens when we drop this additional assumption on G? Krasner achieved considerable success in field theory while using this approach, and it also has been applied to recover semiassociative algebras.

In a talk earlier this year at Bar-Ilan, we laid out categorical foundations for this residue construction, viewed in the more general context of pairs, a setting which permits categorical products, sums, and various extensions. This talk, based on joint work with Jun, deals with one of the trickier aspects, tensor products.

Uriya First:

Title: Involutions of the third kind: counterexamples

Abstract:

Let (A,\tau) be a central simple algebra over a field K, let s:K->K be the restriction of t to K and let F be the fixed subfield of s; we also say that \tau:A->A is an s-involution. Given another CSA with s-involution (A',\tau'), we say that \tau and \tau' are of the same type if (A,\tau) and (A',\tau') become isomorphic after base changing from F to its algebraic closure. Famously, in the case F=K (\tau is of the first kind), there are two types of s-involutions --- orthogonal and symplectic --- and in the case where K/F is quadratic Galois (\tau is of the second kind), there is only one type --- unitary. This classification has a major role in the theory of classical algebraic groups. Over a general (commutative) ring R, the role of CSAs is played by Azumaya algebras. Given an order-2 automorphism s:R->R with fixed subring S, we similarly say that two Azumaya algebras with s-involutions (of the same degree) are of the same type if they become isomorphic after base-changing from S to some faithfully flat extension. As with CSAs, if R=S ("first kind"), then there are two types (if S is connected) --- orthogonal and symplectic ---, and if R/S is quadratic Galois ("second kind"), then there is only one type --- unitary. However, there is a third option in which R \neq S and R/S is not quadratic Galois; call it the "third kind". In this case, we can break Spec(R) into a disjoint union of an open subset U and a closed subset Z

such that R/S is "of the second kind" on U and "of the first kind" on Z. In particular, an s-involution of an Azumaya algebra \tau:A->A can be orthogonal on some points of Z and symplectic on other points, determining a continuous function c(\tau) : Z -> {orthogonal,symplectic}. Some while ago, Ben Williams and I showed that c(\tau) actually determines the type of \tau. I will discuss a recent work with Williams where we demonstrate two peculiarities of the "third kind" case. First, not all functions c(\tau) : Z -> {orthogonal, symplectic} need to arise as types of involutions. Second, there are Azumaya algebras with involution whose type cannot be represented by an Azumaya algebra with a trivial Brauer class.

Pavel Shteyner:

Title: Generalizations of Fountain-Gould quotient rings.

Abstract:

In 1990 Fountain and Gould introduced a new generalization of classical quotient rings based on the notion of group inverse. These new quotient rings have been described for some special classes of rings in subsequent research. In particular, such quotient rings need not have an identity.

The procedure of assigning inverses to certain elements is called localization. It can be carried out, more generally, by considering other generalized inverses, for example, Moore–Penrose, Drazin, and others. In particular, for rings with involution, rings of quotients with respect to Moore–Penrose inverses were studied by Ánh, Márki and Siles Molina. An important feature of the notion introduced by Ánh and Márki is that it is equipped with an additional parameter, namely, one can specify the elements that are required to have inverses. This allows for much more flexibility in dealing with quotient rings.

Recently, the general concept of an inverse along an element which covers and generalizes the notion of outer generalized inverse was introduced and developed by Mary. A similar concept of (b, c)-inverse was developed independently by Drazin. These notions generalize all classical outer inverses and unify many classical notions connected to generalized inverses.

With the help of the inverse along an element, we introduce a new general notion of quotient rings. We show that, on the one hand, this notion unifies all quotient rings constructed using various outer generalized inverses. On the other hand, every such quotient rings is, in fact, a Fountain–Gould quotient ring with an appropriate parameter.

This is joint work with Alexander Guterman and László Márki

Ofir Schnabel

Title: On The Twisted Group Ring Isomorphism Problem

Abstract:

The classical group ring isomorphism problem [GRIP] concerns the following question: given a group G and a commutative ring R, which information about G is encoded in the ring structure of the group ring RG?

In other words how strongly does the representation theory of a group influence its structure?

In this talk we present and study the twisted group ring isomorphism problem [TGRIP] which is a “twisted” variation of the GRIP in the context of twisted group rings or, alternatively, projective representations.

We ask which information about G is encoded in the ring structure of all the twisted group rings of the group G over the ring R, or in other words how

strongly the projective representation theory of a group influences its structure.

We will focus on the cases where R is a field and will deal with:

(1) The differences between the classical problem and its twisted version.

(2) The effect of the ground field on the methods to study TGRIP and on the results obtained. In particular we will deal with cases in which R is the field of complex numbers, R is a finite field and R the field of rational numbers.

This is a joint work with Leo Margolis.

Gil Alon

Title: The structure of algebraic sets over the quaternions

Abstract:

Let R be the ring of polynomials in n commuting variables over the H (=Hamilton's ring of quaternions). Given an n-tuple of quaternions a=(a1,...,an) and a polynomial f in R, the substitution f(a) is canonically defined if a1,...,an commute in pairs. If they do not commute, one has to choose an order on the variables for this substitution to be defined. In an earlier work, we have proved a Nullstellensatz for R, where the relevant geometric space is the set of commuting n-tuples of quaternions. Gori Sarfatti and Vlacci have interpreted this work in the context of slice-regular quaternionic functions and conjectured that for a given order on the variables, a Nullstellensatz for R still holds where the geometric space is H^n. In this talk we will present a proof of the conjecture. No prior knowledge will be assumed. Joint work with Elad Paran.