

Algebraic Geometry section

IMU annual meeting, BGU, July 6, 2021.

Schedule

14:00-14:40	Howard Nuer (Technion) <i>The cohomology of the general stable sheaf on a K3 surface</i>
14:50-15:15	Uriel Sinichkin (TAU) <i>Enumeration of algebraic and tropical singular hypersurfaces</i>
15:15-15:40	Coffee break
15:40-16:20	Eugenii Shustin (TAU) <i>Geometry, topology, and combinatorics of expressive objects</i>
16:30-17:10	Michael Temkin (HUJI) <i>Dream desingularization algorithms</i>

Abstracts

Howard Nuer (Technion)

Title: *The cohomology of the general stable sheaf on a K3 surface*

Abstract: Let X be a K3 surface of Picard rank one and degree $2n$ with ample generator H . Let $M_H(\mathbf{v})$ be the moduli space of Gieseker semistable sheaves on X with Mukai vector \mathbf{v} . In this talk, we consider the weak Brill-Noether property for \mathbf{v} , namely that the general sheaf in $M_H(\mathbf{v})$ has at most one nonzero cohomology group. We show that given any positive rank r , there are only finitely many Mukai vectors of rank r failing weak Brill-Noether over all K3 surfaces of Picard rank one. We discuss our algorithm for finding the potential counterexamples and demonstrate the utility of our approach by discussing how we were able to classify all such counterexamples up to rank 20 and calculate the cohomology of the general sheaf in each case. Moreover, for fixed rank r , we give sharp bounds on n, d and a that guarantee that a Mukai vector $\mathbf{v} = (r, dH, a)$ satisfies weak Brill-Noether. As a corollary, we provide another proof of the classification of Ulrich bundles on K3 surfaces of Picard rank one. In addition, we discuss the question of when the general sheaf in $M_H(\mathbf{v})$ is globally generated. This joint work with Izzet Coskun and Kota Yoshioka makes crucial use of Bridgeland stability conditions and wall-crossing.

Uriel Sinichkin (TAU)

Title: *Enumeration of algebraic and tropical singular hypersurfaces*

Abstract: It is classically known that there exist $(n+1)(d-1)^n$ singular hypersurfaces of degree d in complex projective n -space passing through a prescribed set of points (of the correct size). In this talk I will present an approach to a construction of $\Omega(d^n)$ real singular hypersurfaces through a collection of points in \mathbb{RP}^n . This can be achieved using tropical geometry and can be easily generalized to a construction of hypersurfaces with multiple isolated singularities.

Eugenii Shustin (TAU)

Title: *Geometry, topology, and combinatorics of expressive objects*

Abstract: We survey geometric, topological, and combinatorial features of expressive objects, which are real plane algebraic or analytic curves such that their real topology completely determines the relative topology of the critical point set of the defining polynomial/function. These objects reveal nice real geometric and topological properties (mainly conjectural), and are tightly intertwined with geometry of planar divides, topology of links in the three-sphere, and combinatorics of quivers. The latter relation points to a general conjecture that an expressive object canonically gives rise to a cluster algebra. Time permitting, we quickly indicate higher-dimensional analogues of planar expressive models.

Based on joint works with S. Fomin, P. Pylyavskyy, and D. Thurston.

Michael Temkin (HUJI)

Title: *Dream desingularization algorithms*

Abstract: The most natural attempt to resolve singularities is to define a natural invariant, consider the locus where the invariant is maximal, blow it up, and hope for good... Namely hope that the invariant will decrease, and the iterated procedure will stop after finitely many steps. We call a desingularization algorithm a dream one. For many years it was a common knowledge that canonical resolution of singularities is impossible, but very recently it was independently discovered by McQuillan and Abramovich-Temkin-Włodarczyk that once one allows a wider class of blows ups -- a stack theoretic version of classical weighted blow ups, a natural dream algorithm does exist. Moreover, this is very natural already for the original Hironaka's approach -- the coordinates and the weights defining this blow up were well known for decades. In my talk I will tell about this story.