IMU2021
Operator Algebras Session

July 6, 2021

Schedule

14:00 - 14:30	**Baruch Solel** Technion	Invariant subspaces for certain tuples of operators
14:40 - 15:10	**Guy Salomon** Weizmann	When can a discrete group be decomposed into completely syndetic sets?
15:10 - 15:40	**Coffee**	
15:40 - 16:10	**Orr Shalit** Technion	A von Neumann type inequality for commuting row contractions
16:20 - 16:50	**Ilan Hirshberg** Ben Gurion	Mean cohomological independence dimension and radius of comparison
17:00 - 17:30	**Adam Dor-On** Copenhagen	Finite dimensional approximations and coactions for operator algebras

Titles and Abstracts

Invariant subspaces for certain tuples of operators

Baruch Solel

In this talk I will generalize results of Sarkar and of Bhattacharjee-Eschmeier-Keshari-Sarkar concerning dilations and invariant subspaces for commuting tuples of operators. These authors prove Beurling-Lax-Halmos type results for commuting tuples $T = (T_1, \ldots, T_d)$ of operators that are contractive and pure; that is $\Phi_T(I) \leq I$ and $\Phi_T^n(I) \searrow 0$ where

$$\Phi_T(a) = \sum_i T_i a T_i^*.$$

Here we generalize some of their results to commuting tuples T satisfying similar conditions but for

$$\Phi_T(a) = \Sigma_{\alpha \in \mathbb{F}_d^+} x_{|\alpha|} T_\alpha a T_\alpha^*$$

where $\{x_k\}$ is a sequence of non negative numbers satisfying some natural conditions (where $T_\alpha = T_{\alpha(1)} \cdots T_{\alpha(k)}$ for $k = |\alpha|$). In fact, we deal with a more general situation where each x_k is replaced by a $d^k \times d^k$ matrix.

We also apply these results to subspaces of certain reproducing kernel correspondences E_K (associated with maps-valued kernels K) that are invariant under the multipliers given by the coordinate functions.

When can a discrete group be decomposed into completely syndetic sets?

Guy Salomon

A subset A of a discrete group G is called completely syndetic if for every n there are $g_1, \ldots, g_k \in G$ such that any n elements of G belong together to $g_i A$ for some i.

In this talk, I will discuss the question in the title and present some relations to certain C*-algebras, Boolean algebras and dynamical systems. In particular, I will show how to construct non-trivial minimal proximal actions for non strongly amenable groups. I will also show how this machinery helps to characterize “dense orbit sets” answering a question of Glasner, Tsankov, Weiss, and Zucker.

The talk is based on a joint work with Matthew Kennedy and Sven Raum and an ongoing work with Ariel Yadin.

A von Neumann type inequality for commuting row contractions

Orr Shalit

It is known that there is no von Neumann type inequality for commuting row contractions, in the sense that there is no constant C_d such that

$$\|p(T)\| \leq C_d \sup_{z \in \mathbb{B}_d} |p(z)|$$

for every polynomial p in d commuting variables and every commuting row contraction $T = (T_1, \dots, T_d)$. We show that if one restricts attention to row contractions acting on a space of finite dimension n, then one can obtain such a von Neumann type inequality with a constant $C_d(n)$ that depends also on n. This might seem both obvious and esoteric, but the proof turns out to be subtler than one might guess, and the result turns out to have several interesting consequences for some important operator algebras.

This is joint work in progress with Michael Hartz and Stefan Richter.

Mean cohomological independence dimension and radius of comparison

Ilan Hirshberg

The notions of mean dimension and radius of comparison came about independently and in different contexts. Mean dimension is an invariant for dynamical systems, introduced by Gromov and by Lindenstrauss-Weiss, which has nothing a-priori to do with C^*-algebras. The radius of comparison is an invariant for C^*-algebras defined by Toms, in the context of his work on counterexamples to the Elliott conjecture, which was unrelated to dynamical systems. It appears now that there is a surprising connection between those concepts. In 2010, Giol and Kerr published a construction of a minimal dynamical system whose associated crossed product has positive radius of comparison. Subsequently, Phillips and Toms conjectured that the radius of comparison of a crossed product should be roughly half the mean dimension of the underlying system. Upper bounds for the radius of comparison in terms of mean dimension were obtained by Phillips and by Niu, however there were no general results concerning lower bounds. In the non-dynamical context, work of Elliott and Niu suggests that the right notion of dimension to consider is cohomological dimension, rather than covering dimension (notions which coincide for CW complexes). Motivated by this insight and by the underlying philosophy introduced in Gromov's paper, we introduce an invariant which we call "mean cohomological independence dimension", for actions of countable amenable groups on compact metric spaces, which are related to mean dimension, and obtain general lower bounds for the radius of comparison.

This is joint work with N. Christopher Phillips.

Finite dimensional approximations and coactions for operator algebras

Adam Dor-On

Finite dimensional approximations for all representations of a C*-algebra are available whenever some injective representation has such an approximation. This is a classical result of Exel and Loring from 1992. We extend this paradigm to general (possibly non-self-adjoint) operator algebras.

Our work is intimately related to the question, studied by Clouatre and Ramsey, of whether the maximal C*-cover of an operator algebra is residually-finite dimensional when the algebra itself is. We resolve this question for semigroup operator algebras as well as various algebras of functions, thus providing many previously unattainable examples. A novel key tool in our analysis is the notion of an RFD coaction by a semigroup, whose development uses and extends ideas from the theory of semigroup C*-algebras.

This talk is based on joint work with Raphael Clouatre